2 0 1 1

IT/MCA

Paper : 1.6/3.1

**(Mathematical Foundation of
Computer Science)**

Full Marks : 100

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

SECTION—A

(*Marks* : 50)

Answer **five** questions, including Question Nos. **1**
and **2** which are compulsory

1. Answer the following :   1×10=10

   (a) Give an example of an empty set.

   (b) Let *A* be a set having 5 elements. How
   many nonempty subsets does *A* have?

   (c) Let *A* be a nonempty set and ϕ the
   empty set. What can you say about
   $A \times \phi$?

(2)

(d) Whether the set $\mathbb{Q}$ of all rational numbers is countable or uncountable?

(e) Let $R = \{(1, 1), (2, 2), (3, 3)\}$ be a relation in the set $A = \{1, 2, 3\}$. Is R a transitive relation?

(f) Define the commutative law of statements.

(g) Suppose p stands for 'you work hard' and q stands for 'you will be successful'. Write the verbal translation for $p \Rightarrow q$.

(h) When are two propositions p and q called equivalent?

(i) Let p be the statement, "All students in the college are taking history". Write down $\sim p$.

(j) Write the law of double negation for any statement.

2. Answer the following :　　　　2×5=10

(a) Let $\mathbb{N}$ be the set of natural numbers. Write down the set $\{x : x \in \mathbb{N}$ and $x < 10\}$ in list method.

(b) Find a set A such that
$$\{1, 2\} \subseteq A \subset \{1, 2, 3, 4\}$$

(c) Let $U = \{a, b, c, d, e, f\}$ and $A = \{a, e, f\}$, $B = \{a, d, e\}$. Find $A^c \cap B^c$.

(d) Let p be a statement. Construct the truth table for $\sim p$.

(e) What do you mean by 'existential quantifier'?

3. *(a)* If n subsets of a set are given, how many minsets are generated by them? Let $U = \{1, 2, 3, 4, 5, 6\}$ and let A and B respectively be the subsets $\{1, 3, 4\}$ and $\{1, 4, 6\}$. Find the minsets generated by A and B.   5

(b) For any sets A and B, show that
$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$ 5

4. *(a)* Let $A = \{a, b, c\}$. In A, construct the following :   5

(i) A reflexive relation which is not symmetric

(ii) A relation which is neither reflexive nor symmetric

(iii) An equivalence relation

(b) Prove that any two equivalence classes under an equivalence relation are either identical or disjoint.   5

(4)

5. (a) Using mathematical induction, prove that $8^n - 3^n$ is a multiple of 5.　　5

(b) By construction of truth tables, prove the following :　　5

(i) $\sim (p \vee q) \equiv \sim p \wedge \sim q$

(ii) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

6. (a) Without using truth table, prove that

$$\sim (p \wedge q) \to (\sim p \vee (\sim p \vee q)) \Leftrightarrow (\sim p \vee q)　　5$$

(b) Define Tautology and Contradiction. By a truth table or otherwise, determine if $(p \wedge q) \to (p \vee q)$ is a tautology or not.　　5

SECTION—B

(*Marks* : 50)

Answer **five** questions, including Question Nos. **7** and **8** which are compulsory

7. Answer the following :　　1×10=10

(a) Write down the 2×3 null matrix.

(b) "All diagonal matrices are scalar matrices." State True or False.

(c) "Matrix multiplication is associative." State True or False.

(d) What can be the maximum possible rank of a 3×5 matrix?

(e) What are the eigenvalues of the following matrix?

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(f) Without expansion, determine the value of the determinant

$$\begin{vmatrix} 3 & 0 & 3 \\ 2 & 4 & -2 \\ 1 & -6 & 7 \end{vmatrix}$$

(g) What is simple graph?

(h) What do you mean by a pendant vertex?

(i) Draw a complete graph on two vertices.

(j) What is Eulerian graph?

8. Answer the following : $2 \times 5 = 10$

(a) Give examples of two matrices A and B which are non-zero but AB is a zero matrix.
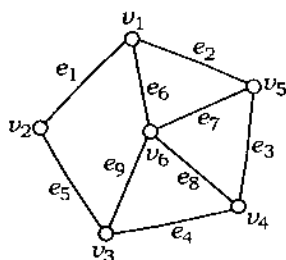
(b) Find the inverse of the matrix

$$\begin{pmatrix} 2 & 2 \\ 3 & 4 \end{pmatrix}$$

(**6**)

(c) Show that

$$\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$$

(d) Define edge cut-set of a connected graph.

(e) Find a vertex cut-set of the following graph :

9. *(a)* Reduce

$$A = \begin{pmatrix} 2 & 2 & 4 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix}$$

to Echelon form. 5

(b) Find the rank of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & 0 & 3 \end{pmatrix}$$

5

(**7**)

10. *(a)* Solve the following system of equations by Cramer's rule : 5
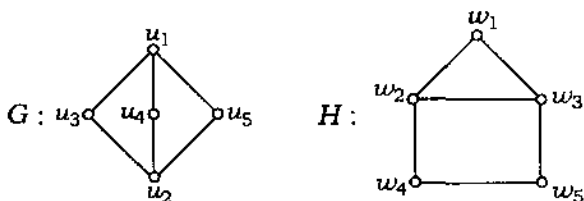
$$x + y + z = 9$$
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$

(b) If X_1 and X_2 are two eigenvectors corresponding to two different eigenvalues λ_1 and λ_2 of a real symmetric matrix A, then show that X_1 and X_2 are orthogonal. 5

11. *(a)* When are two graphs said to be isomorphic? Discuss whether the graphs G and H shown below are isomorphic or not : 5

(b) Prove that a connected graph G has an Eulerian trail if and only if G has exactly two odd vertices. 5

12. *(a)* Define Hamiltonian graph. Let G be a Hamiltonian graph and let S be a nonempty proper subset of the vertex set V. Prove that

$$W(G - S) \leq |S| \qquad 5$$

(b) Define bipartite graph. Prove that a graph is bipartite if and only if it is bichromatic. 5

★ ★ ★